

Probability

1. Complete the following statements:

- (i) Probability of an event E + Probability of the event 'not E ' = _____ .
- (ii) The probability of an event that cannot happen is _____. Such an event is called _____ .
- (iii) The probability of an event that is certain to happen is _____. Such an event is called _____ .
- (iv) The sum of the probabilities of all the elementary events of an experiment is _____ .
- (v) The probability of an event is greater than or equal to _____ and less than or equal to _____ .

Solution:

- (i) Probability of an event E + Probability of the event 'not E ' = **1**.
- (ii) The probability of an event that cannot happen is **0**. Such an event is called **an impossible event**.
- (iii) The probability of an event that is certain to happen is **1**. Such an event is called **a sure or certain event**.
- (iv) The sum of the probabilities of all the elementary events of an experiment is **1**.
- (v) The probability of an event is greater than or equal to **0** and less than or equal to **1**.

2. Which of the following experiments have equally likely outcomes? Explain.

- (i) A driver attempts to start a car. The car starts or does not start.
- (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
- (iii) A trial is made to Solution: a true-false question. The Solution: is right or wrong.
- (iv) A baby is born. It is a boy or a girl.

Solution:

- (i) This statement does not have equally likely outcomes as the car may or may not start depending upon various factors like fuel, etc.
- (ii) Even this statement does not have equally likely outcomes as the player may shoot or miss the shot.
- (iii) This statement has equally likely outcomes as it is known that the solution is either right or wrong.
- (iv) This statement also has equally likely outcomes as it is known that the newly born baby can either be a boy or a girl.

3. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Solution:

Tossing of a coin is a fair way of deciding because the number of possible outcomes are only 2 i.e. either head or tail. Since these two outcomes are an equally likely outcome, tossing is unpredictable and is considered to be completely unbiased.

4. Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

Solution:

The probability of any event (E) always lies between 0 and 1 i.e. $0 \leq P(E) \leq 1$. So, from the above options, option (B) -1.5 cannot be the probability of an event.

5. If $P(E) = 0.05$, what is the probability of 'not E'?

Solution:

We know that,

$$P(E) + P(\text{not } E) = 1$$

It is given that, $P(E) = 0.05$

$$\text{So, } P(\text{not } E) = 1 - P(E)$$

$$\text{Or, } P(\text{not } E) = 1 - 0.05$$

$\therefore P(\text{not } E) = 0.95$

6. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Solution:

(i) We know that the bag only contains lemon-flavoured candies. So, The no. of orange flavoured candies = 0

\therefore The probability of taking out orange flavoured candies = $\frac{0}{1} = 0$

(ii) As there are only lemon flavoured candies, $P(\text{lemon flavoured candies}) = 1$ (or 100%)

7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Solution:

Let the event wherein 2 students having the same birthday be E

Given, $P(E) = 0.992$

We know,

$$P(E) + P(\text{not } E) = 1$$

$$\text{Or, } P(\text{not } E) = 1 - 0.992 = 0.008$$

∴ The probability that the 2 students have the same birthday is 0.008

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is

(i) red?

(ii) not red?

Solution:

The total number of balls = No. of red balls + No. of black balls
So, the total no. of balls = $5 + 3 = 8$

We know that the probability of an event is the ratio between the no. of favourable outcomes and the total number of outcomes.

$$\Rightarrow P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$$

(i) Probability of drawing red balls = $P(\text{red balls}) = (\text{no. of red balls} / \text{total no. of balls}) = 3/8$

(ii) Probability of drawing black balls = $P(\text{black balls}) = (\text{no. of black balls} / \text{total no. of balls}) = 5/8$

9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) red? (ii)

white?

(iii) not green?

Solution:

The Total no. of balls = $5 + 8 + 4 = 17$

$$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$$

(i) Total number of red balls =
 5 P (red ball) = $5/17 = 0.29$

(ii) Total number of white balls =
 8 P (white ball) = $8/17 = 0.47$

(iii) Total number of green balls =
 4 P (green ball) = $4/17 = 0.23$

\therefore P (not green) = $1 - P$ (green ball) = $1 - (4/17) = 0.77$

(Page No: 309)

10. A piggy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins.

If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin?

(ii) will not be a ₹5 coin?

Solution:

Total no. of coins = $100 + 50 + 20 + 10 = 180$

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total number of 50 p coin = 100

$P(50 \text{ p coin}) = 100/180 = 5/9 = 0.55$

(ii) Total number of ₹5 coin = 10

$\therefore P(\text{not ₹5 coin}) = 1 - P(\text{₹5 coin}) = 1 - 0.055 = 0.945$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 15.4). What is the probability that the fish taken out is a male fish?



Fig. 15.4

Solution:

The total number of fish in the tank = $5 + 8 = 13$

Total number of male fish = 5

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

$P(\text{male fish}) = 5/13 = 0.38$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 15.5), and these are equally likely outcomes. What is the probability that it will point at

(i) 8?

(ii) an odd number?

(iii) a number greater than

2? (iv) a number less than 9?

Solution:

Total number of possible outcomes = 8

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total number of favourable events (i.e. 8) = 1

$\therefore P(\text{pointing at } 8) = \frac{1}{8} = 0.125$

(ii) Total number of odd numbers = 4 (1, 3, 5 and 7)

$P(\text{pointing at an odd number}) = \frac{4}{8} = \frac{1}{2} = 0.5$

(iii) Total numbers greater than 2 = 6 (3, 4, 5, 6, 7 and 8)

$P(\text{pointing at a number greater than 4}) = \frac{6}{8} = \frac{3}{4} = 0.75$

(iv) Total numbers less than 9 = 8 (1, 2, 3, 4, 5, 6, 7, and 8)

$P(\text{pointing at a number less than 9}) = \frac{8}{8} = 1$

13. A die is thrown once. Find the probability of getting

(i) a prime number;

(ii) a number lying between 2 and 6;

(iii) an odd number.

Solution:

Total possible events when a dice is thrown = 6 (1, 2, 3, 4, 5, and 6)

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total number of prime numbers = 3 (2, 3 and

5) $P(\text{getting a prime number}) = \frac{3}{6} = \frac{1}{2} = 0.5$

(ii) Total numbers lying between 2 and 6 = 3 (3, 4 and 5) P

(getting a number between 2 and 6) = $\frac{3}{6} = \frac{1}{2} = 0.5$

(iii) Total number of odd numbers = 3 (1, 3 and 5)

$P(\text{getting an odd number}) = \frac{3}{6} = \frac{1}{2} = 0.5$

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red

colour (ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds

Solution:

Total number of possible outcomes = 52

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total numbers of king of red colour = 2
P (getting a king of red colour) = $\frac{2}{52} = \frac{1}{26} = 0.038$

(ii) Total numbers of face cards = 12
P (getting a face card) = $\frac{12}{52} = \frac{3}{13} = 0.23$

(iii) Total numbers of red face cards = 6
P (getting a king of red colour) = $\frac{6}{52} = \frac{3}{26} = 0.11$

(iv) Total numbers of jack of hearts = 1
P (getting a king of red colour) = $\frac{1}{52} = 0.019$

(v) Total numbers of king of spade = 13
P (getting a king of red colour) = $\frac{13}{52} = \frac{1}{4} = 0.25$

(vi) Total numbers of queen of diamonds = 1 P
(getting a king of red colour) = $\frac{1}{52} = 0.019$

15. Five cards the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Solution:

Total numbers of cards = 5

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Numbers of queen = 1

P (picking a queen) = $\frac{1}{5} = 0.2$

(ii) If the queen is drawn and put aside, the total numbers of cards left is $(5 - 1) = 4$

(a) Total numbers of ace = 1

P (picking an ace) = $\frac{1}{4} = 0.25$

(b) Total numbers of queen = 0

P (picking a queen) = $\frac{0}{4} = 0$

16. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Solution:

Numbers of pens = Numbers of defective pens + Numbers of good pens

∴ Total number of pens = 132 + 12 = 144 pens

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

$P(\text{picking a good pen}) = 132/144 = 11/12 = 0.916$

17. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Solution:

(i) Numbers of defective bulbs = 4

The total numbers of bulbs = 20

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

∴ Probability of getting a defective bulb = $P(\text{defective bulb}) = 4/20 = \frac{1}{5} = 0.2$

(ii) Since 1 non-defective bulb is drawn, then the total numbers of bulbs left are

19 So, the total numbers of events (or outcomes) = 19

Numbers of defective bulbs = 19 - 4 = 15

So, the probability that the bulb is not defective = $15/19 = 0.789$

18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

Solution:

The total numbers of discs = 90

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total number of discs having two digit numbers = 81

(Since 1 to 9 are single digit numbers and so, total 2 digit numbers are $90 - 9 = 81$)

P (bearing a two-digit number) = $81/90 = 9/10 = 0.9$

(ii) Total number of perfect square numbers = 9 (1, 4, 9, 16, 25, 36, 49, 64 and

81) P (getting a perfect square number) = $9/90 = 1/10 = 0.1$

(iii) Total numbers which are divisible by 5 = 18 (5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90)

P (getting a number divisible by 5) = $18/90 = \frac{1}{5} = 0.2$

(Page No: 310)

19. A child has a die whose six faces show the letters as given below:

The die is thrown once. What is the probability of getting

(i) A?

(ii) D?

Solution:

The total number of possible outcomes (or events) = 6

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) The total number of faces having A on it = 2

P (getting A) = $2/6 = \frac{1}{3} = 0.33$

(ii) The total number of faces having D on it = 1

P (getting D) = $1/6 = 0.166$

20. Suppose you drop a die at random on the rectangular region shown in Fig. 15.6. What is the probability that it will land inside the circle with diameter 1m?

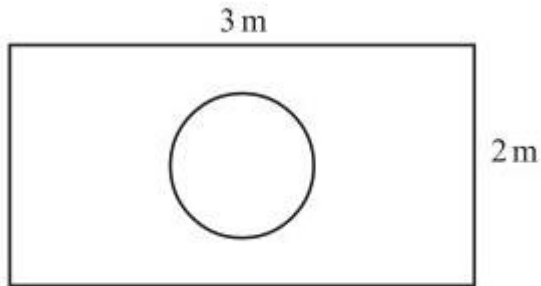


Fig. 15.6

Solution:

First, calculate the area of the rectangle and the area of the circle. Here, the area of the rectangle is the possible outcome and the area of the circle will be the favourable outcome. So, the area of the rectangle = $(3 \times 2) \text{ m}^2 = 6 \text{ m}^2$ and,

The area of the circle = $\pi r^2 = \pi(\frac{1}{2})^2 \text{ m}^2 = \pi/4 \text{ m}^2 = 0.78$

∴ The probability that die will land inside the circle = $\frac{[a]/[A]}{n} = \frac{0.78}{6} = 0.13$

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it?

(ii) She will not buy it?

Solution:

The total numbers of outcomes i.e. pens = 144

Given, numbers of defective pens = 20

∴ The numbers of non defective pens = $144 - 20 = 124$

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

(i) Total numbers events in which she will buy them = 124
So, $P(\text{buying}) = 124/144 = 31/36 = 0.86$

(ii) Total numbers events in which she will not buy them = 20
So, $P(\text{not buying}) = 20/144 = 5/36 = 0.138$

22. Refer to Example 13. (i) Complete the following table:

Event: 'Sum on 2 dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Therefore, each of them has a probability $1/11$. Do you agree with this argument? Justify your Solution:.

Solution:

If 2 dices are thrown, the possible events are:

- (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
- (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
- (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
- (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
- (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
- (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

So, the total numbers of events: $6 \times 6 = 36$

(i) It is given that to get the sum as 2, the probability is $1/36$ as the only possible outcomes = (1,1)

For getting the sum as 3, the possible events (or outcomes) = E (sum 3) = (1,2) and (2,1) So, $P(\text{sum } 3) = 2/36$

Similarly,

E (sum 4) = (1,3), (3,1), and (2,2)

So, $P(\text{sum } 4) = 3/36$

E (sum 5) = (1,4), (4,1), (2,3), and

(3,2) So, $P(\text{sum } 5) = 4/36$

E (sum 6) = (1,5), (5,1), (2,4), (4,2), and (3,3)

So, $P(\text{sum } 6) = 5/36$

E (sum 7) = (1,6), (6,1), (5,2), (2,5), (4,3), and (3,4)

So, $P(\text{sum } 7) = 6/36$

E (sum 8) = (2,6), (6,2), (3,5), (5,3), and (4,4)

So, $P(\text{sum } 8) = 5/36$

E (sum 9) = (3,6), (6,3), (4,5), and

(5,4) So, $P(\text{sum } 9) = 4/36$

E (sum 10) = (4,6), (6,4), and (5,5)

So, $P(\text{sum } 10) = 3/36$

$E(\text{sum } 11) = (5,6), \text{ and } (6,5)$

So, $P(\text{sum } 11) = 2/36$

$E(\text{sum } 12) = (6,6)$

So, $P(\text{sum } 12) = 1/36$

So, the table will be as:

Event: Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(ii) The argument is not correct as it is already justified in (i) that the number of all possible outcomes is 36 and not 11.

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Solution:

The total number of outcomes = 8 (HHH, HHT, HTH, THH, TTH, HTT, THT, TTT)

Total outcomes in which Hanif will lose the game = 6 (HHT, HTH, THH, TTH, HTT, THT)

$P(\text{losing the game}) = 6/8 = \frac{3}{4} = 0.75$

24. A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

[Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]

Solution:

Outcomes are:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

So, the total number of outcome = $6 \times 6 = 36$

(i) Method 1:

\therefore The required probability = $5/6 \times 5/6 = 25/36$

Method 2:

Let E be the event in which 5 does not come up either time.

So, the favourable outcomes are $[36 - (5 + 6)] = 25$

$\therefore P(E) = 25/36$

(ii) Number of events when 5 comes at least once = 11 (5 + 6) \therefore The required probability = $11/36$

(Page No: 311)

25. Which of the following arguments are correct and which are not correct? Give reasons for your Solution:.

(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $1/3$

(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $1/2$

Solution:

(i) All the possible events are (H,H); (H,T); (T,H) and (T,T) So, P (getting two heads) = $1/4$

and, $P(\text{getting one of the each}) = 2/4 = \frac{1}{2}$

∴ This statement is incorrect.

(ii) Since the two outcomes are equally likely, this statement is correct.

(Page No: 311)

Exercise: 15.2

1. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on

(i) the same day?

(ii) consecutive days?

(iii) different days?

Solution:

Since there are 5 days and both can go to the shop in 5 ways each
so, The total number of possible outcomes = $5 \times 5 = 25$

(i) The number of favourable events = 5 (Tue., Tue.), (Wed., Wed.), (Thu., Thu.), (Fri., Fri.), (Sat., Sat.)

So, $P(\text{both visiting on the same day}) = 5/25 = \frac{1}{5}$

(ii) The number of favourable events = 8 (Tue., Wed.), (Wed., Thu.), (Thu., Fri.), (Fri., Sat.), (Sat., Fri.), (Fri., Thu.), (Thu., Wed.), and (Wed., Tue.)

So, $P(\text{both visiting on the consecutive days}) = 8/25$

(iii) $P(\text{both visiting on the different days}) = 1 - P(\text{both visiting on the same day})$

So, $P(\text{both visiting on the different days}) = 1 - \frac{1}{5} = \frac{4}{5}$

2. A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:

		Number in first throw					
+		1	2	2	3	3	6
Number in second throw	1	2	3	3	4	4	7
	2	3	4	4	5	5	8
	2					5	
	3						
	3			5			9
	6	7	8	8	9	9	12

What is the probability that the total score is

(i) even?

(ii) 6?

(iii) at least 6?

Solution:

The table will be as follows:

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

So, the total number of outcome = $6 \times 6 = 36$

(i) E (Even) = 18

P (Even) = $18/36 = \frac{1}{2}$

(ii) E (sum is 6) = 4

P (sum is 6) = $4/36 = 1/9$

(iii) E (sum is at least 6) = 15

P (sum is at least 6) = $15/36 = 5/12$

3. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

Solution:

It is given that the total number of red balls = 5

Let the total number of blue balls = x

So, the total no. of balls = x + 5

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

$\therefore P(\text{drawing a blue ball}) = [x / (x + 5)]$ -----(i)

Similarly,

$P(\text{drawing a red ball}) = [5 / (x + 5)]$ -----(ii)

From equation (i) and (ii)

$x = 10$

So, the total number of blue balls = 10

4. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x

Solution:

Total number of black balls = x

Total number of balls = 12

$P(E) = (\text{Number of favourable outcomes} / \text{Total number of outcomes})$

$P(\text{getting black balls}) = x / 12$ -----(i)

Now, when 6 more black balls are added,

Total balls become = 18

\therefore Total number of black balls = x + 6

Now, $P(\text{getting black balls}) = (x + 6) / 18$ -----(ii)

Solving equation (i) and (ii)

$x = 3$

5. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

Solution:

Total marbles = 24

Let the total green marbles = x

So, the total blue marbles = $24 - x$

$P(\text{getting green marble}) = x/24$

From the question, $x/24 = \frac{2}{3}$

So, the total green marbles = 16

And, the total blue marbles = $24 - x = 8$